A Study of the Stratospheric Major Warming and Subsequent Flow Recovery during the Winter of 1979 with an Isentropic Vertical Coordinate Model

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(Abstrac)
To achieve the goal of this study, we first simulate the entire evolution of the major warming of February and early March 1979. As Manney et al. (1994) did, we use an isentropic vertical coordinate model. A major advantage of isentropic vertical coordinates is that coordinate surfaces coincide with material surfaces under adiabatic conditions. Moreover, Ertel’s potential vorticity for the quasi-static system can be simply expressed as $g(2\Omega \sin \theta + \mathbf{k} \cdot \nabla_p \times \nabla(-\partial_p/\partial\theta)^{-1}$, where $\Omega$ is the earth’s angular velocity, $\varphi$ is latitude, $\mathbf{k}$ is vertical unit vector, and $\nabla$ is horizontal velocity. Since the conservation of Ertel’s potential vorticity plays a central role in the large-scale dynamics of the stratosphere, we expect that isentropic vertical coordinate models like Hsu and Arakawa’s (1990), in which the advantages described above are maintained even after vertical discretization, can simulate the entire evolution of the sudden warming reasonably well.

The simulated Eliassen–Palm (EP) flux and potential vorticity fields, as well as the simulated potential enstrophy budget, are then analyzed. The EP flux has been widely used as an indicator of the propagation of wave activity (Dunkerton et al. 1981; Palmer 1981; Butchart et al. 1982; Palmer and Hsu 1983; Mechoso et al. 1985, 1986). O’Neill and Pope (1988), however, discussed potential limitations of the EP flux approach when westerlies breakdown and flows become highly nonlinear. Potential vorticity field, on the other hand, has been used mainly to describe nonlinear processes without following the traditional practice of separating flow quantities into zonal means and sinusoidal wave components (McIntyre and Palmer 1983; Butchart and Remsberg 1986; Dunkerton and Delisi 1986; Baldwin and Holton 1988; O’Neill and Pope 1988; Manney et al. 1994). Also, potential enstrophy budget has been analyzed for investigating the nonlinear interactions between waves (Smith 1983; Smith et al. 1984; Tao 1994).

In this study, we take the advantage of each approach mentioned above and combine the results of the analyses to understand the key dynamical processes during the entire evolution of the event. We try to minimize the calculation errors in the potential enstrophy budget analysis by using formulations derived from the model’s discrete governing equations. In this way, detailed decomposition of the budget is accomplished without being contaminated by computational errors and inconsistencies between prediction and diagnosis.

The paper is organized as follows. Section 2 reviews the observed features of the February and early March 1979 stratospheric warming. Section 3 describes the isentropic vertical coordinate model used in the study. Sections 4 and 5 present simulations of the warming event with observed and truncated lower boundary conditions, respectively. In these sections, we discuss the model’s performance in simulating the event and the extent to which the evolution of the event is driven by the troposphere. In section 6, the simulated EP flux and eddy potential enstrophy budget are analyzed. Based on these analyses, we discuss the roles of waves 1 and 2 in each stage and the nonlinear amplification of wave 1 in the recovery stage. Finally, section 7 presents summary and conclusions.

2. Observed features of the February 1979 stratospheric major warming

A detailed description of the major warming of February and early March 1979 is given by Andrews et al. (1987 and references therein). Here, we review only basic features in the flow and temperature evolution during the event. The observational dataset is obtained from two sources. Below 10 hPa, we use temperature, geopotential height and wind from the National Centers for Environmental Prediction–National Center for Atmospheric Research (NCEP–NCAR) reanalysis (Kalnay et al. 1996) with a horizontal resolution of $2.5^\circ$ longitude $\times 2.5^\circ$ latitude on the 100-, 70-, 50-, 30-, 20-, and 10-hPa isobaric surfaces. Above 10 hPa, we use temperature and geopotential height fields compiled by the National Meteorological Center (now NCEP) with a horizontal resolution of $5^\circ$ longitude $\times 2.5^\circ$ latitude on the 5-, 2-, 1-, and 0.4-hPa isobaric surfaces. Since winds are not available above 10 hPa, we produce estimates by using the geostrophic relationship except at low latitudes between $20^\circ$N and $20^\circ$S, where a linear interpolation in the meridional direction is used.

Figure 1 shows 10-hPa geopotential height and temperature fields during the warming event. On 17 February, there is a large and elongated cyclone centered near 60$^\circ$E over the polar region and an anticyclone over the Aleutian islands. The lowest temperature is approximately 210 K near the center of the cyclone and the warmest is about 240 K over North America. From 17 to 21 February, the cyclone splits into two, the stronger of which develops over Eurasia and the weaker over North America. An anticyclone forms near 10$^\circ$W over the Atlantic Ocean. From 21 to 25 February, the cyclones separate further and the anticyclones merge across the Pole. A pool of air with temperatures up to about 255 K develops east of the cyclone over Eurasia. From 25 February to 1 March, the cyclone over North America starts weakening whereas the anticyclone in the polar region remains strong. The temperature increases in the entire polar region. After 1 March, the Eurasian cyclone moves toward the Pole where it is joined by the remainder of the North American cyclone. The temperature over the polar region starts decreasing.

In brief, along with a general warming of the polar region, the zonal wave 1 structure of the geopotential height associated with one cyclonic vortex displaced from the Pole evolves into a wave 2 structure with two distinct cyclonic vortices. Later, the wave 2 structure returns to a wave 1 structure as the temperature over the polar region decreases. In view of the flow evolution outlined in this section, we will consider the warming
Fig. 1. Observed (NCEP–NCAR reanalysis) geopotential height (contours) and temperature (grayscale) at 10 hPa, on selected days between 17 Feb and 5 Mar 1979. Projection is polar stereographic with the outer edge at 20°N. The contour interval is 0.2 km and the grayscale is designated on the figure. The thick line represents 30.6-km geopotential height.
and recovery stages of the event separately. These stages are represented by the periods 17–27 February and 28 February–5 March, respectively.

3. Model description

The model we use is a global version of the isentropic vertical coordinate model developed by Hsu and Arakawa (1990). This model predicts the horizontal velocity on isentropic surfaces and the mass between isentropic surfaces. The Montgomery potential is diagnosed from the vertical distribution of mass. The space finite-difference schemes of the model are especially designed to maintain the advantages of isentropic vertical coordinates in modeling, such as 1) the discretization problem for the vertical advection terms is virtually eliminated, and 2) the simple form of potential vorticity makes it easier to maintain its conservation in a vertically discrete system. In particular, potential enstrophy is conserved under adiabatic frictionless processes. The model includes diffusion schemes for layers with very small mass.

a. Governing equations

The governing equations written in spherical coordinates are

$$\frac{\partial u}{\partial t} - qv^* + \frac{1}{a \cos \varphi} \frac{\partial}{\partial \lambda} (K + M) + \frac{\partial}{\partial \varphi} \frac{\partial u}{\partial \varphi} = 0, \hspace{1cm} (1)$$

$$\frac{\partial v}{\partial t} + qu^* + \frac{1}{a \cos \varphi} (K + M) + \frac{\partial}{\partial \varphi} \frac{\partial v}{\partial \varphi} = 0, \hspace{1cm} (2)$$

$$\frac{\partial m}{\partial t} + \frac{1}{a \cos \varphi} \frac{\partial M}{\partial \lambda}$$

$$+ \frac{1}{a \cos \varphi} \frac{\partial}{\partial \varphi} (v^* \cos \varphi) + \frac{\partial}{\partial \varphi} (m \theta) = 0, \hspace{1cm} (3)$$

$$\frac{\partial M}{\partial \theta} = \Pi, \hspace{1cm} (4)$$

$$\dot{\theta} = Q \Pi, \hspace{1cm} (5)$$

where $\lambda$ is longitude, $\varphi$ is latitude, and $a$ is earth’s radius. Further, $q$ is potential vorticity defined by

$$q = \frac{1}{a \cos \varphi} \left[ \frac{\partial}{\partial \lambda} (u \cos \varphi) - \frac{\partial}{\partial \varphi} \left( f + \frac{1}{a \cos \varphi} \left( \frac{\partial}{\partial \lambda} - \frac{\partial}{\partial \varphi} (u \cos \varphi) \right) \right) \right], \hspace{1cm} (6)$$

where

$$m = -a \frac{\partial p}{\partial \theta}. \hspace{1cm} (7)$$

In addition,

$$(u^*, v^*) = (mu, mv); \hspace{1cm} (8)$$

$$K = \frac{1}{2} (u^2 + v^2); \hspace{1cm} (9)$$

$M$ is the Montgomery potential defined by

$$M = c_T \Phi + \Pi; \hspace{1cm} (10)$$

$\Pi$ is the Exner function,

$$\Pi = c_p (p/p_0)^{\gamma}; \hspace{1cm} (11)$$

and $Q$ is the diabatic heating rate.

b. Upper and lower boundary conditions

The vertical model domain covers the stratosphere above the 400-K surface. For the simulation, which will be referred to as “control,” the Montgomery potential is prescribed on the lower boundary. In the hypothesis-testing experiments (see section 5), we use a special lower boundary condition to minimize spurious reflections of downward propagating waves. Further details on this lower boundary condition are given in section 5. At the upper and lower boundaries, we assume that there is no diabatic heating ($Q = 0$).

To reduce spurious reflections of upward propagating waves at the upper boundary, we use a Rayleigh-type friction in the top four model layers, with coefficients increasing upward. For the top layer, the coefficient is $2.8 \times 10^{-6} \text{ s}^{-1}$.

c. Spatial and time discretization

The vertical discretization of the model follows Hsu and Arakawa (1990). The horizontal velocity and mass are predicted for the layers and the vertical mass flux representing diabatic heating is calculated at the interfaces. The horizontal discretization is based on Arakawa and Hsu (1990) generalized to the spherical coordinate system following Arakawa and Lamb (1981). The time discretization follows Konor and Arakawa (1997).

d. Additional model features

To avoid the generation of artificially strong winds in layers with small mass, we include the vertical momentum diffusion scheme of Konor and Arakawa (1997). This diffusion acts only when the thickness of the layer becomes very small.

At this stage of development, the model does not include comprehensive physical processes. Instead, the thermal forcing is given in the form of a Newtonian-type heating in which pressure is relaxed to its zonally averaged initial value with a timescale of about 20 days in the lower stratosphere and about 5 days in the upper
stratosphere. (Recall that, in the $\theta$ coordinate, the space distribution of pressure determines the thermal structure of the model atmosphere.) This formulation is a crude representation of radiative processes; yet it is useful for the objective of this paper.

e. Model resolution

Preliminary experiments suggested that model results are less sensitive to horizontal than to vertical resolution. For the simulation presented here, the horizontal resolution is $5^\circ$ longitude $\times 4^\circ$ latitude. We use a 30-s time step, which is short enough to maintain linear computational stability without any zonal filtering near the Poles. There are 23 model layers, spaced nearly evenly in height, resulting in a vertical resolution of about 1.7 km except for the highest layer whose depth is approximately 0.7 hPa. The potential temperatures of the isentropic layers are 414, 444, 479, 518, 561, 607, 654, 710, 770, 831, 901, 974, 1058, 1160, 1263, 1358, 1470, 1600, 1724, 1858, 2005, 2145, and 2688 K.

4. Simulation of the February 1979 stratospheric major warming

The model integration starts on 17 February 1979 and is 20 days long to cover the entire evolution of the warming event. We obtain initial pressures on the model’s isentropic surfaces by linear interpolation using observed temperatures on pressure levels. The initial horizontal velocities are also obtained by linear interpolation from those observed. [Even though the initial fields are obtained from two data sources (see section 2), we have not found any sign of inconsistency in the simulation results.] During the integration, the Montgomery potential is prescribed at the lower boundary using a linear time interpolation from the daily NCEP–NCAR reanalysis.

Figure 2 shows a selection of simulated 10-hPa geopotential height and temperature fields. A comparison of Fig. 2 with Fig. 1 indicates that the evolution of geopotential height is realistically simulated. The complete splitting of the cyclone over the polar region and its later recovery are both captured. The locations of cyclones and anticyclones are also well simulated, although simulated cyclones are slightly weak. The simulated temperature field shows a realistic evolution of the warming, especially east of the cyclone over Eurasia and in the polar region. However, the temperature gradient near the center of the Eurasian cyclone is somewhat weak. Additionally, the simulated rate of temperature decrease over the polar region after 1 March is smaller than observed. Yet the model’s success in simulating the flow recovery after the warming with such a relatively coarse vertical resolution is encouraging. Manney et al. (1994) attributed their similar success to the use of an isentropic vertical coordinate.

Figure 3 shows meridional cross sections of observed and simulated zonal-mean zonal wind on 17 (initial state) and 25 February (warming stage), and on 5 March (recovery stage). A comparison with the observations indicates that the breakdown of westerlies and the acceleration of easterlies in high latitudes are well simulated. On 5 March, the simulated westerlies also resemble those observed in the lower stratosphere, but in the upper stratosphere simulated winds are still easterly. The existence of an effective upper boundary due to the use of a thick top layer is a likely contributor to this error in the upper stratosphere.

The evolution of Rossby–Ertel potential vorticity (PV = $gg$; $g = 9.8 \text{ m s}^{-2}$) on the 831-K isentropic surface from the observation and the simulation is shown in Fig. 4. On 25 February, we can see clearly an elongated tongue of high-PV air that appears to be pulled out of the vortex over North America. This high-PV tongue and associated small-scale features seem to show the main features of planetary-wave breaking (McIntyre and Palmer 1983). Meanwhile, low-PV air is advected from low latitudes into the polar region east of the Eurasian vortex. The low-PV tongue approximately coincides with the region of temperature increase shown in Fig. 1c, indicating that the increase is associated with air descend. By 5 March, the Eurasian vortex with a reduced intensity is relocated across the Pole. The close agreement with the observation shows that the model captures major features in the PV evolution through the warming and recovery stages, such as the complete breakdown of the initial vortex over the polar region into two vortices, the advection of low-PV air from low to high latitudes, and the recovery of the high-PV vortex over the polar region.

In view of these results, we conclude that the model’s simulation of the warming and subsequent flow recovery can be used as the basis for hypothesis-testing experiments. In this regard, the results obtained so far will be labeled as “control” in the remainder of the paper.

5. Hypothesis-testing experiments

In order to better understand the extent to which the wave forcing at the lower boundary is important for the warming and subsequent flow recovery, we perform two hypothesis-testing experiments using truncated initial and lower boundary conditions (see Table 1). The initial conditions for both experiments consist of the zonal mean and the wave 1 and 2 components of the velocity and pressure fields. The Montgomery potential at the lower boundary in experiment I consists of the zonal mean and waves 1 and 2, while that in experiment II consists only of the zonal mean and wave 2. As in control, the zonal-mean component at the lower boundary corresponds to that observed. The wave components at the lower boundary, on the other hand, are not always taken from observations as a revised lower boundary condition is used to minimize spurious reflections of downward propagating waves. The revised condition is
Fig. 2. As Fig. 1 but for simulated geopotential height and temperature.
Fig. 3. Meridional cross sections of observed (a), (b), and (d) and simulated (c) and (e) zonal-mean zonal wind, on selected days. Westerlies and easterlies are shown as thick solid lines and dotted lines, respectively. The contour interval is 10 m s⁻¹. Thin horizontal solid lines show isentropic surfaces that correspond to the vertical levels of the model for the simulations.

based on the direction of vertical propagation of wave activity, which is inferred by inspection of the phase tilt of the wave in the Montgomery potential field between the lowest two model layers at each latitude. If the direction is upward—that is, if the tilt is westward—the Montgomery potential for the wave component at the lower boundary exactly corresponds to that observed. Otherwise, the component is obtained through extrap-
Fig. 4. Evolution of the Rossby–Ertel potential vorticity on the 831-K isentropic surface obtained from observations (a) and (b) and the simulation (c). Projection is polar stereographic with an inner circle of 60°N and outer edge at 20°N. The contour interval is 50 potential vorticity units (PVU = 10^{-6} \text{ m}^2 \text{ K kg}^{-1} \text{ s}^{-1}). The regions of low potential vorticity (less than 200 PVU) are shaded.
In simulating the recovery stage, the amplification of decay of wave 2 is slow after day 9, especially in the later amplification hardly exists. We also note that the condition takes place during the first few days but the experiment II, the amplification of wave 1 in the initial periods, one between day 0 and day 4, and the other between day 10 and day 17. On the contrary, wave 2 tilts westward with height during the entire simulation period. These features appear throughout high latitudes although we show only the cross sections at 70°N. As we discuss in section 6a, a westward (eastward) tilt with height normally implies upward (downward) propagation of wave activity. Thus, it seems that wave 2 always propagates upward; on the contrary, wave 1 propagates upward before day 10 and downward afterward.

These results suggest that the early amplification of waves 1 and 2 in the stratosphere is induced by the upward propagation of their counterparts in the troposphere. The later amplification of wave 1, however, must be due to a mechanism within the stratosphere since the wave is propagating downward. This means that the existence of wave 1 at the lower boundary during that stage does not matter for the evolution of wave 1 above. Then, the failure of experiment II in simulating the recovery stage suggests that the amplification of wave 1 during the recovery stage needs the existence of wave 1 at the lower boundary during the preceding stage.

In the following section, we further examine the behavior of waves 1 and 2 during the warming and recovery stages.

### 6. Analyses of experiment I

In this section we analyze the results obtained in experiment I in more detail since it produces a realistic simulation in a framework simpler than control. Our main goal is to understand the roles of waves 1 and 2 in the evolution of the event and the mechanism for the later amplification of wave 1. To facilitate the analysis, we separate the entire simulation period into three stages based on the evolution of wave 1 (Fig. 6a). They will be referred to as stage I (days 0–4), stage II (days 5–9), and stage III (days 10–17). Roughly, stage I and stage II correspond to the period of warming and stage III corresponds to the recovery. We first examine the EP flux for waves 1 and 2 during the entire evolution. Then, we focus on the eddy potential enstrophy budget to obtain insights into the amplification of wave 1 during stage III.

#### a. Eliassen–Palm flux

The EP flux based on linear wave theory is a useful tool for diagnosing the forcing of the mean flow by
waves. As Edmon et al. (1980) pointed out, the EP flux can be also used as an indicator of the propagation of wave activity. When the meridional gradient of the zonal-mean (quasigeostrophic) potential vorticity is positive, as is normally the case, the direction of the EP flux is the same as that of wave activity propagation. Dunkerton et al. (1981), Palmer (1981), and Butchart et al. (1982) are good examples of studies on stratospheric sudden warmings that use the EP flux as a diagnostic.

Here, we use the EP flux formalism generalized to the primitive equation system with the \( \theta \) coordinate, which is given by Andrews (1983) and Andrews et al. (1987). Nevertheless, we display the EP flux, \( \mathbf{F} = (F_{\phi}, F_{\varphi}) \), and its divergence, \( \nabla \cdot \mathbf{F} \), on \( (\varphi, z) \) plots, in which \( z = -H \ln(p/p_s) \), where \( H \) is a scale height (7 km) and \( p_s \) is a standard reference pressure (1000 hPa), following two steps. First, we interpolate \( (F_{\phi}, F_{\varphi}) \) to a grid with uniform spacing in both \( \varphi \) and \( z \) to obtain \( (F'_{\phi}, F'_{\varphi}) \). Second, we scale the interpolated values in the following way:

![Figure 5](image-url)

**Fig. 5.** Simulated geopotential height (contours) and temperature (grayscale) at 10 hPa, from EXP I (left) and EXP II (right). Upper panels are for the warming stage, on 21 Feb, and lower panels are for the recovery stage, on 5 Mar. Figure arrangement is as in Fig. 1.
\[
\begin{align*}
\mathbf{F}_w &= C_w \times F_z'(w), \\
\mathbf{F}_z &= C_z \times F_z'(z),
\end{align*}
\]  

(12)

Here, \( C_w \) and \( C_z \) are used to eliminate the geometric distortion inherent in the use of different vertical and latitudinal scales. In the following diagrams, \( C_w = 0.009 \) and \( C_z \) is the area mean of \( z/u \), which takes into account the variations of \( u \) with height.

For waves 1 and 2, Fig. 8 shows averages over each stage of \( \mathbf{F} = (\mathbf{F}_w, \mathbf{F}_z) \) and \( \text{DF} \) given by

\[
\text{DF} = \frac{g}{m} \frac{1}{\cos \phi} \nabla \cdot \mathbf{F},
\]  

(13)

which is the mean zonal force per unit mass by waves. In stage I, the EP fluxes of both waves 1 and 2 are upward. Since the meridional gradient of the zonal-mean potential vorticity is positive everywhere in the domain, this indicates that wave activity propagates upward for both waves 1 and 2. The contours of \( \text{DF} \) show a convergence of the EP flux in high latitudes also for both waves 1 and 2. In particular, there is polar focused convergence of the wave 1 flux. This convergence is consistent with the deceleration of westerlies and the development of easterlies during the warming stage (Fig. 3c). During stage II, the EP fluxes of both waves 1 and 2 are still upward but are focused equatorward. The contours of \( \text{DF} \) for wave 2 show almost the same convergence pattern as in stage I. For wave 1, however, the contours of \( \text{DF} \) show a more expanded divergence pattern between 70° and 60°N than in stage I. During stage III, on the other hand, waves 1 and 2 behave in a completely different way. The wave 1 EP fluxes are downward in high latitudes, while those of wave 2 are still upward. Since the meridional gradient of the potential vorticity is positive in the lower stratosphere, this indicates that wave 1 propagates downward from the midstratosphere, while wave 2 still propagates upward from the lower boundary. This result implies that the stage...
III amplification of wave 1 is due to a mechanism within the stratosphere. In the same stage, the contours of DF show a divergence pattern for wave 1 and a convergence pattern for wave 2. Since westerlies are accelerated in high latitudes in stage III (Fig. 3e), it appears that wave 1 plays the main role in the flow recovery, competing with the deceleration effect on westerlies by wave 2.

b. Eddy potential enstrophy budget

The eddy potential enstrophy budget provides a theoretical framework for investigating the potential enstrophy sources and sinks associated with waves. Smith (1983) and Smith et al. (1984) discussed the relative importance of each process contributing to the local change of potential enstrophy of planetary waves. Tao (1994) performed similar analysis with an isentropic coordinate model. Both noted, however, that the budget analysis involving high-order derivatives was very sensitive to data accuracy and calculation errors. Since we use an isentropic vertical coordinate model that maintains conservation of total potential enstrophy in a discrete system under adiabatic and frictionless processes, we have a computational advantage in this analysis.

The zonal-mean eddy potential enstrophy equation with an isentropic coordinate is given by

\[
m \frac{\partial}{\partial t} \left( \frac{q^*}{2} \right) = -\frac{\nu^*}{a} \frac{1}{\partial \varphi} \left( \frac{q^*}{2} \right) - \frac{q^* v^{*w}}{a} \frac{1}{\partial \varphi} + mD + m\overline{q^*}N_{\lambda},
\]

where

\[
m\overline{q^*}N_{\lambda} = -q^* \left( \frac{1}{a \cos \varphi} \frac{\partial q^*}{\partial \lambda} + \frac{1}{a} \frac{\partial q^*}{\partial \varphi} \right).
\]

See the appendix for the derivation of (14) and the expression for the diabatic effects, \(mD\). In this equation, the local time change of zonal-mean eddy potential enstrophy, which essentially represents the wave activity, is due to advection by the mean meridional flow, linear conversion (wave–mean flow interaction), diabatic effects, and nonlinear transport and conversion (wave–wave interaction).

Since we focus on the time evolutions of waves 1 and 2, we further derive the eddy potential enstrophy equation for each wave component. Multiplying the eddy potential vorticity equation (A6) by \(q^*\) for waves 1, 2, and the remainder of the waves and zonally averaging the result, we obtain the corresponding zonal-mean eddy potential enstrophy equations,

\[
m \frac{\partial}{\partial t} \left( \frac{q_{i(1)}^*/2}{2} \right) = -\nu^* q_{i(1)}^* \frac{1}{\partial \varphi} - \frac{q_{i(1)}^* v^{*w}}{a} \frac{1}{\partial \varphi} + mD + m\overline{q_{i(1)}^*}N_{\lambda},
\]

where the subscript \(n\) represents 1, 2, and 3. The subscripts (1), (2), and (3+) denote zonal waves 1, 2, and wavenumbers 3 and higher, respectively. The nonlinear terms of (16) are identified as

\[
m \overline{q_{i(1)}^*}N_{\lambda} = \frac{-1}{a} \frac{\partial}{\partial \varphi} \left( \frac{a^* q_{i(1)}^*}{2} \right) + \frac{C_{12}}{2} + \frac{C_{13+}}{2},
\]

\[
m \overline{q_{i(2)}^*}N_{\lambda} = \frac{-1}{a} \frac{\partial}{\partial \varphi} \left( \frac{a^* q_{i(2)}^*}{2} \right) + \frac{C_{12}}{2} + \frac{C_{13+}}{2},
\]

\[
m \overline{q_{i(3+)}^*}N_{\lambda} = \frac{-1}{a} \frac{\partial}{\partial \varphi} \left( \frac{a^* q_{i(3+)}^*}{2} \right) + \frac{C_{12}}{2} + \frac{C_{13+}}{2}.
\]
Fig. 8. Meridional cross sections of the EP flux $\mathbf{F}$ and the eddy forcing $DF$ for waves 1 (left) and 2 (right) averaged over each stage. Upper, middle, and lower panels represent stages I, II, and III, respectively. Arrows are EP fluxes and their scales are specified on each frame in units of kg m s$^{-2}$ K$^{-1}$. Thin and thick solid contours in the figure show the convergence and the divergence of EP fluxes, respectively; the contour interval is 2 m s$^{-1}$ day$^{-1}$. Thick dash-dot lines show $\tilde{\zeta} = 0$, thus the region inside of the line (which is shaded gray) has negative mean potential vorticity gradient.
\[ m q_{13} N = - \frac{1}{a} \frac{\partial}{\partial \phi} \left[ \left( u^{\prime} q_{13}^3 \right) - \left( q_{13}^2 u^{\prime} q_{13}^1 \right) - \left( q_{13} q_{13} q_{13}^3 \right) \right] \]
\[ - \frac{C_{13}}{2} - C_{23}, \quad (19) \]

where we assume that the mass flux is nondivergent. The quantities \( C_{12}, C_{13}, \) and \( C_{23} \) are defined by

\[ C_m = - \left[ q_i^{13} \left( u^{\prime} \frac{1}{a} \frac{\partial q_{im}^1}{\partial \lambda} + u^{\prime} \frac{1}{a} \frac{\partial q_{im}^1}{\partial \phi} \right) \right. \]
\[ - q_{im} \left( u^{\prime} \frac{1}{a} \frac{\partial q_{1m}^1}{\partial \lambda} + u^{\prime} \frac{1}{a} \frac{\partial q_{1m}^1}{\partial \phi} \right), \quad (20) \]

where \( n = 1 \) or 2 and \( m = 2 \) or 3+. In (17)–(19), we regard the terms associated with \( v^{\prime} \) as the nonlinear transport and \( C_{12}, C_{13}, \) and \( C_{23} \) as the nonlinear conversion terms between waves specified with subscripts (1), (2), and (3+).

We first calculated each term in (14) using the finite-difference expressions derived from the model’s discrete governing equations. Then, we examined the residual of (14) to see how well the balance is maintained in the discrete system. In this test, we obtained the local time change of zonal-mean eddy potential enstrophy by running the model for several time steps. The result was then compared with the change determined from the right-hand side of the equation. We found this residual to be negligible almost everywhere in the domain throughout the entire simulation period. Based on this calculation accuracy, we perform detailed decomposition of the potential enstrophy budget as in (16). In the following we focus on stage III and only show the effects of the linear conversion and nonlinear terms of the budget since those of mean meridional advection and diabatic source/sink turn out to be relatively small.

Figure 9 shows latitude–height sections of the dominant terms in (16) averaged over stage III. During this stage, the processes associated with wave 1 are dominant. Through linear conversion, wave 1 loses potential enstrophy to the mean flow. This loss is consistent with the acceleration of mean westerlies and the result of the EP flux analysis for wave 1 (Fig. 8e). Through nonlinear processes, however, the potential enstrophy of wave 1 significantly increases in high latitudes. Such an increase more than compensates the loss of potential enstrophy to the mean flow, resulting in the intensification of wave 1 activity near the polar region. For wave 2, on the other hand, the gain from the mean flow is comparable to the loss through nonlinear processes, and hence the wave 2 activity hardly changes.

The strong net tendency to increase the potential enstrophy of wave 1 near the polar region between layers 5 and 15 (23–41 km) (see Fig. 9e) corresponds well to the amplification of this wave component during the same stage. Primarily, the net tendency is due to the nonlinear processes as described above. To further investigate this tendency, we separate the nonlinear terms in (16) to transport and conversion processes following (17)–(20). Figure 10 shows those effects for waves 1 and 2 (the sum of the three effects represents the nonlinear effect shown in Figs. 9c,d). It appears that through nonlinear conversion wave 1 gains potential enstrophy from wave 2 and loses a part of it to waves with higher wavenumbers. The large loss of potential enstrophy of wave 2 into wave 1 is almost compensated by the gain from waves with higher wavenumbers and that by transport. From these, we infer that the potential enstrophy transition from waves with higher wavenumbers into wave 1 via wave 2 (“anticascade”) should be attributed to the significant tendency to increase of wave 1 shown in Fig. 9.

This anticascade of potential enstrophy is associated with the poleward extension of small-scale features in the potential vorticity field accompanied by the Eurasian vortex. To further elaborate this point, we show the potential vorticity fields on the 1160 K isentropic surface in Fig. 11 on selected days. The PV structure of wave 2 on day 4 (stage I) suggests that the Eurasian vortex captures the low-PV air from low latitudes and elongates it toward and across the polar region (day 8). During stage III, as the low-PV tongue further extends poleward, its leading part is cut off (days 12 and 14). This cutoff part of the low-PV tongue and the high-PV Eurasian vortex inside the 60°N latitudinal circle form a wave 1 structure (day 14). In longitude, the low-PV tongue is a high zonal wavenumber feature in low latitudes while it is a low zonal wavenumber feature in high latitudes. The poleward extension of low-PV tongue associated with the Eurasian vortex appears as an anticascade of potential enstrophy in the budget analysis.

7. Summary and conclusions

In February and early March 1979 the northern stratosphere experienced a major warming associated with a breakdown and subsequent recovery of the polar night vortex. A number of studies on the prewarming, onset and maturity stages of the warming event have been performed using observational data, model simulations and numerical forecasts. The principal goal of this paper is to gain further insight into the dynamical processes during the warming event, with a special emphasis on the recovery stage. To achieve this goal, we numerically simulated the entire evolution of the warming event and then quantitatively analyzed the results.

For the numerical simulation, we used an isentropic vertical coordinate model that is a generalization of Hsu and Arakawa (1990) to the global domain. The discretization used in this model maintains most of the advantages of isentropic vertical coordinates. In particular, the model conserves total potential enstrophy under adiabatic frictionless conditions. The vertical domain of the model covers the stratosphere above the 400-K isentropic surface.
Fig. 9. Meridional cross sections of major terms in the eddy potential enstrophy budget for waves 1 (left) and 2 (right) averaged over stage III: linear conversion between waves and mean flow (a) and (b), nonlinear transport and local conversion between waves (c) and (d), and sum of both (e) and (f). The ordinate represents the model layers, whose representative heights are given on the right side of the frames. Thin and thick solid contours in the figure show negative and positive values, respectively; the contour interval is $4 \times 10^{-16}$ s$^{-1}$. 
Fig. 10. Meridional cross sections of terms representing the nonlinear transport and local conversion for waves 1 (left) and 2 (right): nonlinear transport (a) and (b), conversion between waves 1 and 2 (c) and (d), conversion between waves 1 and 3+ (e), and conversion between waves 2 and 3+ (f). The contour interval is $4 \times 10^{-16} \, \text{s}^{-3}$. 
Physical processes are represented by a simple Newtonian-type heating or cooling. NCEP–NCAR datasets were used for initial and boundary conditions. Integrations started on 17 February 1979 and were 20 days long to cover the entire evolution of the warming event.

When the initial and lower boundary conditions were taken from observations, the model produced a realistic evolution of the major warming. We believe that the successful simulation of the event, especially that of the vortex recovery after the warming period, can be attributed to use of the isentropic vertical coordinate model in which potential vorticity dynamics can be quite accurately represented.

Analyses of the simulated evolution of waves 1 and 2 showed that wave 1 went through two amplification periods, one during the warming stage and the other during the recovery stage. Wave 2, on the other hand, was amplifying during the warming stage and decaying afterward. Based on two hypothesis-testing experiments using truncated initial and lower boundary conditions, we conclude that the early amplification of waves 1 and 2 is induced by upward propagation from the troposphere, but the later amplification of wave 1 takes place through a mechanism within the stratosphere. During the recovery stage, therefore, wave 1 at the lower boundary hardly affects the stratospheric flow above. (Wave 1 forcing at the lower boundary during the preceding stage, however, plays an important role in the later amplification of wave 1 and flow recovery through producing the proper evolution of the stratospheric flow.) This suggests that the dynamics of the in situ amplification of wave 1 is closely related to that of the flow recovery. The importance of wave 1 in the flow recovery can be seen in the simulated EP flux field, which shows that wave 1 plays a primary role in the mean-flow transition from easterlies to westerlies.

In order to investigate the in situ amplification mechanism of wave 1 during the recovery stage, we analyzed the eddy potential enstrophy budget. This analysis requires elaborate computational procedures since it involves highly derived quantities so that the result can be very sensitive to calculation errors. We tried to minimize...
these errors by using a formulation derived from the model’s discrete governing equations, instead of using one directly discretized from the continuous form of the eddy potential enstrophy equation. The results show that, during the recovery stage, wave 1 amplifies by gaining potential enstrophy from waves with higher zonal wavenumbers through nonlinear conversions. We also point out that such an “anticascade” of potential enstrophy from waves with higher zonal wavenumbers into wave 1 is associated with the poleward intrusion of the Eurasian vortex and an accompanying tongue of low PV.

One of the main points of the paper is to relate different ways of viewing the same phenomena: from the points of view of wave effects on the mean flow, potential enstrophy conversion and transport, and potential vorticity redistribution on a synoptic chart. The vortex recovery stage of the major warming event, which is characterized by a wave 1 structure, is described from these complementary viewpoints.

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APPENDIX

Derivation of the Zonal-Mean Eddy Potential Enstrophy Equation

With the spherical coordinates, the potential vorticity and potential enstrophy equations can be written as

\[
m \frac{\partial q}{\partial t} + u^* \frac{1}{a \cos \phi} \frac{\partial q}{\partial \lambda} + v^* \frac{1}{a \partial \phi} \frac{\partial q}{\partial \phi} = q \frac{\partial}{\partial \theta} (m \frac{\partial \theta}{\partial \lambda}) + \left( u^* \frac{1}{a \cos \phi} \frac{\partial q}{\partial \lambda} + v^* \frac{1}{a \partial \phi} \frac{\partial q}{\partial \phi} \right)
\]

(A1)

\[
m \frac{\partial (q^2)}{\partial t} + u^* \frac{1}{a \cos \phi} \frac{\partial (q^2)}{\partial \lambda} + v^* \frac{1}{a \partial \phi} \frac{\partial (q^2)}{\partial \phi} = q^2 \frac{\partial}{\partial \theta} (m \frac{\partial \theta}{\partial \lambda}) + \left( u^* \frac{1}{a \cos \phi} \frac{\partial (q^2)}{\partial \lambda} + v^* \frac{1}{a \partial \phi} \frac{\partial (q^2)}{\partial \phi} \right)
\]

(A2)

where \( \lambda \) is longitude, \( \phi \) is latitude, \( a \) is earth’s radius, \( q \) is potential vorticity, \( m = - \partial \theta / \partial \lambda \), and \((u^*, v^*) = (mu, mv))\). To derive the eddy potential vorticity and eddy potential enstrophy equations, we divide the variables into their zonal means and eddy components as

\[
\begin{align*}
    u^* &= \bar{u}^* + u^*; \\
    v^* &= \bar{v}^* + v^*; \\
    m &= \bar{m} + m^\prime; \\
    \theta &= \bar{\theta} + \theta^\prime; \\
    q &= \bar{q} + q^\prime
\end{align*}
\]

(A3)

where

\[
\bar{A} = \frac{1}{2\pi} \int_0^{2\pi} A \, d\lambda \quad \text{and} \quad A' = A - \bar{A}.
\]

(A4)

Substituting (A3) into (A1) and using the approximations

\[
\begin{align*}
    \bar{u}^* &= \bar{u}^*; \\
    \bar{v}^* &= \bar{v}^*; \\
    \bar{m} &= \bar{m}; \\
    \bar{\theta} &= \bar{\theta}^\prime; \\
    \bar{q} &= \bar{q}^\prime
\end{align*}
\]

(A5)

we can obtain the eddy potential vorticity equation given by

\[
\begin{align*}
    \bar{m} \frac{\partial q^\prime}{\partial t} + u^* \frac{1}{a \cos \phi} \frac{\partial q^\prime}{\partial \lambda} + v^* \frac{1}{a \partial \phi} \frac{\partial q^\prime}{\partial \phi} &= q^\prime \frac{\partial}{\partial \theta} (\bar{m} \frac{\partial \theta}{\partial \lambda}) + \left( u^* \frac{1}{a \cos \phi} \frac{\partial q^\prime}{\partial \lambda} + v^* \frac{1}{a \partial \phi} \frac{\partial q^\prime}{\partial \phi} \right)
\end{align*}
\]

(A6)

where \( N = N_\lambda + N_\theta \).
where
\[ \bar{m} \frac{\partial}{\partial \theta} \left( \frac{q^2}{2} \right) = -\bar{u} * \frac{1}{a \cos \varphi} \frac{\partial q}{\partial \lambda} + \bar{u} * \frac{1}{a \cos \varphi} \frac{\partial q'}{\partial \varphi} + \bar{m} \bar{D} \]

Multiplying (A6) by \( q' \) and zonal averaging the result, we obtain the zonal-mean eddy potential enstrophy equation
\[ \bar{m} \frac{\partial}{\partial \theta} \left( \frac{q'^2}{2} \right) = -\bar{u} * \frac{1}{a \cos \varphi} \frac{\partial q'}{\partial \lambda} + \bar{u} * \frac{1}{a \cos \varphi} \frac{\partial q'}{\partial \varphi} + \bar{m} \bar{D} \]

\[ + \bar{m} q' N, \quad (A8) \]

where
\[ \bar{m} = \bar{q} \left[ \frac{\partial}{\partial \theta} \left( \bar{m} \bar{\theta}' \right) \right] + \frac{1}{a \cos \varphi} \left[ \frac{\partial}{\partial \varphi} \left( \bar{u} \cos \varphi \right) \right] - \frac{\partial}{\partial \theta} \left( \bar{u} \cos \varphi \right) + \bar{m} q' N. \quad (A9) \]

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