Background
The goal of cumulus cloud parameterization is to realize changes in the simulated large-scale environment as a function of the collective influence of multiple cumulus clouds, thereby gaining computational efficiency by being able to operate atmospheric models at larger than cloud-resolving scale resolution. This is often accomplished by utilizing radiative-convective quasi-equilibrium (RCE) whereby, for example, increases in convective available potential energy (CAPE) brought about by upper-level radiative cooling and surface evaporation are assumed to be in approximate balance with CAPE-reducing warming and drying caused by large-scale subsidence induced by cumulus convection.

The desire to create stochastic convective parameterizations (SCPs) has developed from the realization that RCE-based (or otherwise diagnostic and deterministic) convective parameterizations fail to reproduce the full spectrum of convective variability, when employed in global circulation models (GCMs) with lower spatial resolution—this is found, mainly at the small-scale, in CRM ensembles and observational data. For example, sufficiently large amplitude convective heating variations exhibit a departure from RCE that may temporarily stop convection, thus removing constraints of RCE. Such intermittent departures from RCE are inherent to convection, and the use of SCPs has the effect of interrupting RCE and thereby corrects the variability of the convection.

Implementation of SCPs can be as simple as introducing a random multiplicative variable in a given parameterization to increase overall ensemble spread and improve probabilistic precipitation forecasts, but such an approach is not a true physical parameterization, directly linked to resolved processes. A more complex, yet physically based, method requires an understanding of the nature of the deviation from RCE to be able to direct convective variability on a more informed manner. Stochastically adding in the previously lacking variability is a task which will need to take into account a number of parameters, including the chosen grid spacing and the time scale of the large-scale forcing.

Objectives
This study sought to explore the following:

- Under constant large-scale forcing, how does the convective response deviate from a variety of applied forcings?
- Does a CRM under constant large-scale forcing match well with the expected RCE convective response?

Methods
To obtain a characterization of "true" convective variability, the three-dimensional Jung-Arnalan anelastic cloud-resolving model (CRM), which uses the vector flux notation in its dynamical core, is used in this study. Convective statistics were compiled using the model with a 2-km horizontal resolution and a 20-level stretched vertical grid (+20 km) placed at an observational dataset. A doubly periodic grid covering the domain of (255 km) on an F-plane at 15 degrees North latitude was used. The simulations were initialized with a GATE-II sounding containing moderate vertical wind shear.

A number of simulations were performed to study the non-equilibrium, stochastic component of moist convective heating and drying. Following Xu et al. (1992), the response of the non-deterministic component of the numerical simulations is tested by means of 13 simulations using cubic prescribed large-scale forcings with periods ranging from 2 to 120 hours. As a function of time, the periodic forcing follows the form:

\[ F(t) = \left( 1 - \exp \left( -\frac{t}{T} \right) \right) \]

where \( T \) is the period of the time variation. Each of the periodic forcing simulations were run to a length of 10 cycles, representing 15 realizations of the same event. Statistics of the composite of the cycles are heavily relied upon. The dependence of the simulation characteristics on the size of the computational domain was investigated by sub-sampling the full domain. Additionally, 10, 3-day simulations were run at increments of 10% of the maximum large-scale forcing from the periodic simulations.

Characteristics of six variables were used to describe the convective activity: surface precipitation, cloud fraction, cloud mass flux through a box of non-precipitating condensate, and horizontal and vertical eddy kinetic energy in both raw and moist-normalized form.

Convective Response to Constant Forcing

In each of the panels above, the domain-averaged surface precipitation is shown. The number in the title of each panel denotes a percent of the prescribed large-scale forcing shown in the previous box. When the model is run at constant forcing, the response is close to RCE than for the case of variable forcing.

By composing the 15 cycles shown in the box to the left, most of the non-RCE scatter-like response can be averaged out, as shown by the solid black curves in the above figure. Each of the simulations on the left is representative of a different length periodic forcing. As the forcing period decreases, the response is more out of phase with the forcing in a relative sense, but as an absolute sense, the forcing leads to the lead response by ~40 minutes. For a short period forcing, it is difficult for the convection to keep pace. Though it is difficult to see here, the variability of the response about the mean tends to decrease slightly with increasing length of the forcing period.

On the right are composites of the cloud mass flux response through the 3-km layer for various sub-sections of the domain for a forcing with 30-hour periodicity. As in the previous case, the response lags the forcing by ~40 minutes. Here we can observe tendencies for convective response to increase with decreasing domain size. For this variable, the change in variability with domain size appears gradual, with a possible step between 1/16th of the 256 by 256 km domain and a quarter of the domain. In the case of the precipitation response, there is a very sharp shift in the variability of the convective response between one-half of the total domain and the total domain.

Variation with Period Length and Domain Size

The thick solid black line in the plots above represents the maximum of the correlation response to the prescribed forcing. The variability around RCE is itself variable with dependences on multiple parameters. It varies most strongly with changes in the size of the CRM computational domain (e.g. grid size in a GCM). It also varies strongly with forcing, as expected, and is at a lesser extent with changes in the forcing period length.

Differences Between Increasing/Decreasing Forcing

In fact, the mean domain-averaged surface precipitation equilibrium response increases linearly with increasing constant forcing. Standard deviations of the same parameter also increase with increasing forcing, and as the figure on the right shows, the variability of the forcing scales with the mean convective response. This observation is important to note in development of a stochastic convective parameterization.