Development of Global Cloud Resolving Model

Tests with 3D elliptic solver

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Development of a 3D poisson solver

- Based on Arakawa, Jung and Konor
- The continuous equation

\[ \nabla^2 w + \frac{\partial}{\partial z} \left[ \frac{1}{\rho} \frac{\partial}{\partial z} (\rho w) \right] = rhs \]

- The discrete equation

\[
\frac{1}{A_i} \sum_{i'} \frac{w_{i',k+1/2}^{(\kappa)} - w_{i,k+1/2}^{(\kappa+1)}}{L_{i,i'}} L_{i;i'} l_{i,i'} \\
\frac{1}{\delta z_{k+1/2}} \left[ \frac{1}{\rho_{k+1/2} \delta z_{k+1}} \left( \rho_{k+3/2} w_{k+3/2}^{(\kappa+1)} - \rho_{k+1/2} w_{k+1/2}^{(\kappa+1)} \right) - \frac{1}{\rho_k \delta z_k} \left( \rho_{k+1/2} w_{k+1/2}^{(\kappa+1)} - \rho_{k-1/2} w_{k-1/2}^{(\kappa+1)} \right) \right] = rhs_{i,k+1/2}
\]

where \((\kappa)\) denotes an iteration index.
Development of a 3D poisson solver

- Re-arrange to form an implicit tridiagonal system in the vertical

\[
\frac{\rho_{k-1/2}}{\delta z_{k+1/2}} w_{k-1/2}^{(\kappa+1)} = \left[ \frac{1}{A_i} \sum_{i'} l_{i,i'+i'} \frac{L_{i;i+i'}}{\delta z_{k+1/2}} + \frac{\rho_{k+1/2}}{\delta z_{k+1/2}} \left( \frac{1}{\rho_{k+1} \delta z_{k+1}} + \frac{1}{\rho_k \delta z_k} \right) \right] w_{i,k+1/2}^{(\kappa+1)} + \frac{\rho_{k+3/2}}{\delta z_{k+1/2} \rho_{k+1} \delta z_{k+1}} w_{k+3/2}^{(\kappa+1)}
\]

\[
= rhs_{i,k+1/2} - \frac{1}{A_i} \sum_{i'} l_{i,i'+i'} w_{i+i',k+1/2}^{(\kappa)}
\]

- A straightforward modification of the relaxation operator within the 2D multigrid.

- With the current domain decomposition the entire vertical column is local information. Scaling results of the 2D solver will indicate the scaling of the 3D solver.
Convergence of 3D-multigrid -- analytic test 3

- Solve the Poisson equation:
  \[ \nabla^2 w + \frac{\partial}{\partial z} \left[ \frac{1}{\rho} \frac{\partial}{\partial z} (\rho w) \right] = rhs \]

- We can make-up an analytic test case and numerically differentiate to form rhs
  \[ w(\lambda, \varphi, z) = 4 \sin(4\lambda) \cos^2(5\varphi) \sin^3\left(8\pi \frac{z}{z_T}\right) \]

- Infinity-norm of the difference between the true and numerical as a function of V-cycles

- 40 layers
Franklin (2D multigrid, 20 V-cycles, 20 layers)

The NERSC Cray XT4 system, named Franklin, is a massively parallel processing (MPP) system with 9,660 compute nodes. Each node has quad processor cores, and the entire system has a total of 38,640 processor cores.

Each compute nodes consists of a 2.3 GHz single socket quad-core AMD Opteron processor (Budapest) with a theoretical peak performance of 9.2 GFlop/sec per core (4 flops/cycle if using SSE128 instructions). Each compute node has 8 GB of memory (2 GB of memory per core), and each service node (e.g. login node) has 8 GB of memory. Each compute node is connected to a dedicated SeaStar2 router through Hypertransport with a 3D torus topology.

<table>
<thead>
<tr>
<th>Grid resolution</th>
<th>Number of cores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (s)</td>
<td>640</td>
</tr>
<tr>
<td>2,621,442 (9) (15.64km)</td>
<td>0.563</td>
</tr>
<tr>
<td>10,485,762 (10) (7.819km)</td>
<td>2.306</td>
</tr>
<tr>
<td>41,943,042 (11) (3.909km)</td>
<td>9.434</td>
</tr>
<tr>
<td>167,088,642 (12) (1.955km)</td>
<td>insufficient memory per core</td>
</tr>
<tr>
<td>671,088,642 (13) (0.977km)</td>
<td>insufficient memory per core</td>
</tr>
</tbody>
</table>
Current status and work

✦ Much of the computational infrastructure of the baroclinic model can be reused. e.g. parallel domain decomposition and communication

✦ Parallel 3D multigrid methods work well for elliptic equations on the icosahedral grid. 2D multigrid scales well to large numbers of processes.

✦ The stretching and tilting terms in the vorticity equations are straightforward and use many of the grid metrics developed for other operators.

✦ Advection is defined at cell centers, corners and edges:
  - Centers. 3rd-order upstream biased. Done
  - Corners. 3rd-order upstream biased. Done
  - Edges. Currently being developed.